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UNIVERSITY OF TORONTO
COMP. STUDY

ONTARIO
DEPARTMENT
OF
EDUCATION

SENIOR DIVISION

MATHEMATICS

GRADES 11, 12, 13

CURRICULUM S.12

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ONTARIO

MATHEMATICS

SENIOR DIVISION
GRADES 11, 12 and 13

This pamphlet combines the contents of Curriculum S.12, printed in 1957, and Curriculum S.13, printed in 1955.

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COURSES OF STUDY

MATHEMATICS

For

Grades 11, 12 and 13

In

Secondary Schools (General Course)

General Observations

1. The courses in Algebra and Geometry outlined below are the result of the experimental work which has been carried on in certain classes of Grades 11 and 12 during the school years 1950-51 and 1951-52 in order to explore the possibility of distributing more equitably over Grades 11, 12 and 13 the weight of prescribed work in the courses leading to the Grade 13 Departmental Examinations in Mathematics.

2. It is the intention that the modifications which have been made in the existing courses shall lighten to some extent the programme in Grade 13 Algebra and shall result in a more adequate preparation for the study of Trigonometry and Statics. It is also hoped that greater attention to graphs will facilitate later work in Analytic Geometry.

3. Pupils who have completed successfully the general course Mathematics of Grades 9 and 10 should be able to cope with the Grades 11 and 12 courses as revised. Further, with the development of the programme of options in the various grades, the number of pupils attempting these courses without an adequate foundation should progressively decrease.

4. It will be noted that certain items in the courses are designated as "optional". It is important that these be included in the work taken with above-average classes or in the assignments given to the better students within a class-group of varying ability. In other words, it is recommended not only that the assignment should be on an extended rather than a daily basis, but that

the technique of graded or minimum and maximum, assignments should be followed and that the more difficult examples or those which stress merely manipulative dexterity should usually be assigned only to the abler students and reviewed with them on an individual or a group, rather than a class, basis. It goes without saying that all students must, however, reach a suitable minimum standard in order to be recommended for their graduation diplomas.

5. An appreciable amount of class time may be saved, particularly on the material of sections 1 to 3 of the Algebra and also throughout the course in Geometry, by the procedure suggested in the foregoing paragraphs and by the adjustments proposed under "suggestions for Teachers" on page 10.

6. The order in which the courses in Algebra and Geometry are taken in Grades 11 and 12 has been left, in the past, to the discretion of the local authorities. A memorandum (1957-58:37) dated June 2, 1958, requested all schools, for an experimental period of five years, to adopt the plan of taking Geometry in Grade 11 and Algebra in Grade 12. This order is accordingly followed in the arrangement of the syllabus.

7. The contents of the pamphlet entitled "Suggestions for Teachers of Geometry" are reprinted here, following the Grade 11 course.

GRADE 11 GEOMETRY

The work of Grade 10 reviewed and extended to include the following propositions, together with deductions based thereon, and a discussion of the topic of ratio, proportion and variation with geometric applications:

Lines parallel to the same straight line are parallel to each other.

The straight line which joins the middle points of two sides of a triangle is parallel to the third side and equal to one-half of it.

The straight line drawn through the middle point of one side of a triangle parallel to another side bisects the third side.

To divide a given straight line into any number of equal parts.

The sum of the angles of a polygon of n sides is (2n—4) right angles.

If two sides of one triangle are respectively equal to two sides of another triangle, and the angles opposite one pair of equal sides are equal, the angles opposite the other pair of equal sides are either equal or supplementary.

If two equal triangles are on the same side of a common base, the straight line which joins their vertices is parallel to the base.

The complements of the parallelograms about the diagonal of a parallelogram are equal.

To construct a parallelogram equal to a given triangle and having an angle equal to a given angle.

To construct a triangle equal to a given rectilineal figure.

To construct a triangle equal to a given triangle, and having one of its sides equal to a given straight line.

The square on the sum of two straight lines is equal to the sum of the squares on the two lines increased by twice the rectangle contained by them. (Algebraic proof only.)

The square on the difference of two straight lines is equal to the sum of the squares on the lines, diminished by twice the rectangle contained by them. (Algebraic proof only.)

The difference of the squares on two straight lines is equal to the rectangle contained by the sum and the difference of the two lines. (Algebraic proof only.)

If the square on one side of a triangle is equal to the squares on the other two sides, the angle contained by these two sides is a right angle.

The locus of points on one side of a given straight line and at a given distance from that line is a straight line parallel to the given line, through any point at the given distance from the given line.

The locus of a point which is equidistant from two fixed points is the right bisector of the straight line joining the two points.

The locus of points which are equidistant from two given intersecting straight lines is the pair of lines which bisect the angles between the given lines.

The right bisector of any chord of a circle passes through the centre.

To find the centre of a given circle.

To circumscribe a circle about a triangle.

Chords of a circle that are equally distant from the centre are equal.

Equal chords in a circle are equally distant from the centre.

Of two chords in a circle, the one which is nearer the centre is greater than the one which is more remote.

Of two unequal chords in a circle, the greater is nearer the centre than the less.

An angle at the centre of a circle is double an angle at the circumference standing on the same arc.

Angles in the same segment of a circle are equal.

If the straight line joining two points subtends equal angles at two other points, on the same side of it, the four points are concyclic.

The angle in a semicircle is a right angle.

The circle described on the hypotenuse of a right-angled triangle as diameter passes through the vertex of the right angle.

The opposite angles of a quadrilateral inscribed in a circle are supplementary.

If a pair of opposite angles of a quadrilateral are supplementary, its vertices are concyclic.

The straight line drawn perpendicular to a radius of a circle, at the circumference, is a tangent.

To draw a tangent to a circle from a given point outside the circle.

If two tangents are drawn to a circle from an external point, (1) the two tangents are equal, (2) they subtend equal angles at the centre, (3) they make equal angles with the straight line joining the point to the centre.

To inscribe a circle in a triangle.

To draw the escribed circles of a triangle.

To inscribe a circle in a given regular polygon.

About a circle to circumscribe a triangle, equiangular to a given triangle.

If from the point of contact of a tangent to a circle a chord be drawn, each of the angles between the chord and the tangent is equal to the angle in the segment on the other side of the chord.

On a given straight line, to describe a segment of a circle, containing an angle equal to a given angle.

In a given circle, to inscribe a triangle equiangular to a given triangle.

If two circles touch each other, their line of centres passes through the point of contact.

Arcs of a circle which subtend equal angles at the centre are equal.

The areas of triangles of equal altitude are proportional to their bases.

A straight line drawn parallel to one side of a triangle divides the other two sides proportionally.

If two sides of a triangle are divided in the same ratio, the straight line joining the points of section is parallel to the third side.

To find the fourth proportional to three given straight lines.

To divide a given straight line (1) internally, (2) externally, in a given ratio.

If two triangles are equiangular to each other, their

corresponding sides are proportional and hence the triangles are similar.

If two triangles have the sides of one proportional to the sides of the other, they are equiangular to each other, and hence are similar.

If two triangles have an angle of one equal to an angle of the other, and the sides about these angles proportional, the triangles are similar.

The areas of similar triangles are proportional to the squares on corresponding sides.

The perpendicular to the hypotenuse of a right-angled triangle from the opposite vertex is a mean proportional between the segments of the hypotenuse, and each of the sides about the right angle is a mean proportional between the hypotenuse and the adjacent segment of the hypotenuse.

To find a straight line which is a mean proportional between two given straight lines.

To construct a square equal in area to a given rectilineal figure.

If two chords of a circle intersect, the rectangle contained by the segments of one is equal to the rectangle contained by the segments of the other.

If two straight lines intersect so that the rectangle contained by the segments of one is equal to the rectangle contained by the segments of the other, the ends of the lines are concyclic.

If from a point outside a circle a tangent be drawn to the circle, and also a secant, the square on the tangent is equal to the rectangle contained by the secant and the part of it outside the circle.

If from a point outside a circle, two straight lines be drawn, one of which meets the circle, and the other is a secant, and if the square on the line which meets the circle is equal to the rectangle contained by the secant and the part of it outside the circle, the line which meets the circle is a tangent.

The bisector of any angle of a triangle divides the opposite side into segments which have the same ratio as the sides about that angle.

If one side of a triangle is divided into segments which

are proportional to the other two sides, the straight line which joins the point of section to the opposite vertex bisects the angle at that vertex.

The bisector of the exterior angle at any vertex of a triangle divides the opposite side externally into segments which have the same ratio as the other two sides.

If one side of a triangle is divided externally into segments which are proportional to the other two sides, the straight line which joins the point of section to the opposite vertex bisects the exterior angle at that vertex.

In equal circles, angles at the centre are proportional to the arcs on which they stand.

On a given straight line to construct a polygon similar to a given polygon.

Similar polygons can be divided into similar triangles.

The areas of similar polygons are proportional to the squares on corresponding sides.

In a right-angled triangle, any rectilineal figure described on the hypotenuse is equal to the sum of the similar and similarly described figures on the other two sides.

In an obtuse-angled triangle, the square on the side opposite the obtuse angle is equal to the sum of the squares on the sides containing the obtuse angle, increased by twice the rectangle contained by either of these sides and the projection of the other side on it.

In any triangle, the square on the side opposite an acute angle is equal to the sum of the squares on the sides containing that angle, diminished by twice the rectangle contained by one of these sides and the projection of the other side on it.

In any triangle, the sum of the squares on two sides is equal to twice the square on half the third side increased by twice the square on the mediam to the third side.

Suggestions for Teachers: Grade 11 Geometry

Note: The following material was formerly printed separately in Curriculum S. 12B.

I. **General Suggestions:**

1. Where algebraic proofs are employed—and they are valuable in supplementing as well as replacing geometric proofs—care should be taken to maintain the geometric concept.
2. Stress analysis, the use of clear, precise English, the nature of proof, and applications of geometric reasoning to life situations.
3. Divide the exercises, supplementing them where possible and advisable, into three sections:
   (a) arithmetical and algebraic applications
   (b) deductions, as in the present text, but arranged in order of difficulty
   (c) harder deductions for the more able students.
4. Supplement the deductions based on a particular theorem or pair of theorems with sets of exercises of a more comprehensive character, for the solution of which the pupil must keep in mind all the propositions dealt with up to that point.

II. **Comments on the Details of the Course:**

"A Modern Geometry for High Schools"

Re Book I:

1. Proposition 10 might well be introduced after proposition 7 and proposition 28 after 16.
2. It should be noted that propositions 30 and 31 are omitted.
3. It is suggested that the topic of "Locus" be introduced at the end of Book I. The discussion should include propositions 16 and 17 of Book II and also that dealing with the locus of points at a given distance from a given straight line.

   The plotting of a course and the mapping of a property from given bearings and distances are interesting applications of this topic and form a useful preliminary to similar work in Trigonometry.
4. The solution of deductions by analysis should be dealt with more fully than is indicated by the text-

book treatment. In fact, the method of analysis should continually be employed.

Re Book II:

1. The proofs of propositions 2 and 7 need not be stressed for examination purposes.
2. The topic of "Projections" and propositions 8, 9, 10 of Book V may well be taken immediately after the theorem of Pythagoras and its converse.

Re Book III:

The proofs for propositions 6, 7, 20, 21 and 24 need not be stressed for examination purposes.

Re Book IV:

1. Propositions 20 and 23 may well be taken immediately after proposition 3.
2. It should be noted that propositions 17 to 19 and 24 are omitted from the regular course of study. However, they might well be drawn to the attention of the more able students.
3. It is important that the topic of "Ratio, Proportion and Elementary Variation" be made an integral part of the work on Book IV and that its algebraic treatment be incorporated in the study of similar triangles. Certain suggestions regarding the treatment of the topic are given in Section IV.
4. As a practical application of the concept of ratio and introduction to the trigonometry of Grade 13, the teacher should at this stage deal with the fundamental trigonometric ratios and give the pupils some practice in their use in the solution of right-angled triangles.

III. Ratio, Proportion and Variation:

It is essential that the algebraic concepts and the geometrical applications be correlated as intimately as possible.

Definition of ratio, proportion, continued proportion, mean proportional, third proportional.

The following results with related geometric applications:

$\frac{a}{b} = \frac{ma}{mb}$; if $\frac{a}{b} = \frac{c}{d}$ then $ad = bc$, and

$$\frac{b}{a}=\frac{d}{c};\quad \frac{a}{c}=\frac{b}{d};\quad \text{also } a = bK \text{ and } c = dK;$$

$$\text{if } \frac{a}{b}=\frac{c}{d} \quad \text{then } \frac{a}{a+b}=\frac{c}{c+d} \quad \text{etc.}$$

$$\frac{a}{b}=\frac{c}{d}=\frac{e}{f}=\text{etc.}=\frac{a+c+e+}{b+d+f+}$$

Elementary variation — direct, inverse, and simple concrete examples of joint variation. The depth of treatment is suggested in the following section.

IV. Suggestions Regarding the Treatment of Elementary Variation:

The topic of "Variation" gives an excellent opportunity for rounding out the work in ratio and proportion, for studying the function of an independent variable, for giving practice in handling formulae, and for illustrating these various aspects of the same basic idea by the graphical method. Further, the mathematical applications are straightforward and practical, and the manipulative work is comparatively simple.

1. In reviewing the theorem "The areas of triangles of equal altitude are proportional to their bases," the pupil is led to see that this fact may be expressed in a different form—"The area of a triangle of constant altitude varies directly as its base." This involves, of course, the discussion of the meaning of the expression "vary directly." The emphasis here, of course, should be on the idea that, as the base increases or decreases in any given ratio, the area of the triangle increases or decreases in the same ratio.

$$\frac{A_2}{A_1}=\frac{b_2}{b_1} \quad \text{if h is a constant}$$

This should be accompanied by numerous illustrations such as: The circumference of a circle varies as the diameter; number of miles travelled at con-

stant speed varies as the time in hours; wages paid a worker varies as the number of hours he is employed; current in a given circuit varies as the applied voltage; pressure of a constant volume of gas varies as the absolute temperature; the area of a circle varies as the square of the radius; the time of swing of a pendulum varies as the square root of its length; etc.

One of the best of these illustrations to use in bringing out the idea of the constant variation is that involving the circumference of a circle. A discussion leads to the idea that since:

$$\frac{C_1}{C_2} = \frac{D_1}{D_2} \quad . \quad . \quad \frac{C_1}{D_1} = \frac{C_2}{D_2} \quad \text{etc.} = K. \text{ Hence } C = KD$$

and K is really the number π. Discuss some of the other illustrations similarly.

2. The next step would naturally be to extend the idea to inverse variation, where $x \propto 1/y$. Examples might be:

   The time for a journey varies inversely as the speed; the current obtained from a battery varies inversely as the resistance of the circuit; the volume of a given mass of gas at constant temperature varies inversely as the pressure; the frequency of vibration of a wire varies inversely as its length; the intensity of heat radiation varies inversely as the square of the distance from the source, etc. The class should be assigned a number of numerical examples on each of these phases of the work.

3. We may now go back to the original example and discuss, in an elementary way, the subject of joint variation.

   If we double the base of a triangle of constant altitude, we double its area. If we now treble its altitude, we treble the new area and the final area is six times the original. Hence we gradually develop the idea that $A \propto bh$ or $A = Kbh$. From the familiar formula we recognize that $K = 1/2$.

   Use other illustrations such as: Distance travelled depends on time and average speed; wages earned depend on number of hours and rate of pay;

volume of cylinder depends on height and area of base; etc. It is important to emphasize the idea that practically all formulae summarize one or more variation concepts.

4. Two important aspects of this topic remain. They may be introduced concurrently with the above but may best, perhaps, be used to review and fix the foregoing ideas.

   If $C \propto D$ or $C = \pi D$, then C is a function of D. D may be considered the independent, and C the dependent, variable. In general, if $y \propto x$, y is a function of x, — about the simplest possible example of the function concept. Further, all these functions may readily be illustrated graphically. Thus the examples

$$C = 2\pi r, \quad A = \pi r^2, \quad V = \frac{4}{3}\pi r^3$$

   might well be plotted on the same pair of axes to portray the varying rates of increase in the dependent variables as r increases.

   In a similar way, it is valuable to study the graph of an example of inverse variation—such as $pv = K$—with or without the introduction of the name "rectangular hyperbola."

5. Finally, a brief reference might be made to such cases of variation as $y = K_1 + K_2x$ with such simple practical examples as, — the cost of printing programmes for a concert consists of two items, one a constant (setting up the type, etc.) and the other varying directly as the number of copies (cost of paper, labour, etc.). Only simple uninvolved examples need be taken.

GRADE 12 ALGEBRA

Section 1. **Algebraic Notation**

Review of the four fundamental operations.

Extension of multiplication and division to include binomial and trinomial multipliers and divisors. Problems involving fractional and literal coefficients.

Inexact division. Verification and checking.

Practice in the use of general numbers.

Section 2. **Equations of the First Degree**

Review of equations of the first degree in one unknown and extension to include equations with literal coefficients.

Equations of the first degree in two unknowns, including those with literal coefficients.

Graphs of linear functions and graphical solutions of pairs of linear equations in two unknowns.

It is important that the idea of function be introduced early and gradually developed throughout this and the Grade 13 course.

Equations of the first degree in three unknowns.

Problems solved by means of equations of the first degree.

Section 3. **Factoring and Its Applications**

Review of all types of factoring, with oral and written practice on straight-forward examples arranged in miscellaneous collections. The use of class time on artificial or unduly complicated examples should be avoided.

Use of the Factor Theorem in factoring such expressions as

$x^3+5x^2-2x-24$.

Application of factoring to finding H.C.F. and L.C.M.

Optional—Finding of H.C.F. by the method of differences, limited to polynomials of degree 3 or less.

Operations with fractions. Excessive manipulative work is to be avoided. Simplification of complex fractions (of reasonable difficulty), by multiplication of numerator and denominator by L.C.M. of individual denominators, should be emphasized.

Equations involving fractions, their simplification

and solution. Undue emphasis on specialized forms of fractional equations should be avoided.

Section 4. **Powers, Roots and Surds**

Formation, by inspection, of the terms in the expansion of such expressions as $(x+2y-3z)^2$, $(x^2+2x+3)^2$.

Finding, by inspection, square roots of trinomials which are complete squares; also of such other squares as $a^2+9b^2+4c^2-6ab+4ac-12bc$.

Formal method of finding square roots of algebraic expressions which are perfect squares. Calculation of square roots of numbers.

Quadratic surds. Distinction between rational numbers and surds.

Use of Pythagorean theorem to construct lines of length 22″, 23″, 25″, etc.

Operations with quadratic surds, including rationalizing of denominators.

The square-root table.

Solution of equations with surd coefficients.

Problems, involving surds, on the perimeters and areas of triangles and polygons.

Use of the formula for the area of a triangle in terms of the sides.

Section 5. **Ratio, Proportion, and Elementary Variation**

It is expected that the major portion of the work on these topics will have been taken in Grade 11 in conjunction with the work on similar triangles in the course in Geometry.

Definition of ratio, proportion, mean proportional.

The following results, with related exercises:

aK, bK will represent any two numbers in the ratio a: b;

$$\frac{a}{b}=\frac{ma}{mb}; \qquad \text{if } \frac{a}{b}=\frac{c}{d} \text{ then } ad=bc, \qquad \frac{b}{a}=\frac{d}{c}, \ \frac{a}{c}=\frac{b}{d};$$

also $a=bK$ and $c=dK$.

Meaning of direct, inverse and joint variation, with easy problems.

Note: Problems designed merely to give practice in

manipulation should be treated as optional. The objective is to give the pupils familiarity with the principles of ratio, proportion and variation so that they may be able to apply them to problems in Algebra, Physics and Chemistry.

Section 6. **Quadratic Equations**

Examples of quadratic equations with numerical and with literal coefficients.

Solution by factoring and by completing the square.

Solution of the general quadratic equation.

Solution, by formula, of quadratic equations possessing real roots.

Graphs of quadratic functions of one unknown as illustrative of the function and as a method of determining the roots of the corresponding equation.

Graded problems leading to quadratic equations.

Section 7. **Quadratic Equations in Two Unknowns**

Solution of a system of two equations of which one is linear and one is quadratic.

Solution of such systems of two quadratic equations as are reducible to systems of the preceding type.

Graphical solution of a pair of equations in two unknowns, one of which is linear and one quadratic, particularly the circle, parabola and rectangular hyperbola.

Problems leading to systems of equations of the types of this section.

Section 8. **Theory of Quadratics**

The discriminant of a quadratic equation; its use in determining the existence or non-existence of real roots; condition for equal roots. The sum and the product of the roots of a quadratic equation; their use as a check on the roots, and in forming an equation with given roots.

Optional: 1. Symmetrical functions of the roots.

2. Problems involving the obtaining of an equation with roots related to those of a given equation.

Section 9. **Indices**

Index laws for positive integral indices.

Definition and use of powers with fractional, zero and negative indices.

Section 10. **Logarithms**

Definition of logarithm.

Use of tables for finding the logarithm of a given number and for finding the number having a given logarithm.

Use of logarithms for calculating products, quotients, powers and roots.

Problems leading to computations in which the use of logarithms is advantageous. Extensive and continuing practice is desirable.

Solution of such equations as $3^x=80$.

Optional: The Slide Rule.

Section 11. **Surds and Surd Equations**

Further examples on quadratic surds.

Surds of higher order than the quadratic. Simple examples.

Simple surd equations resulting in linear or quadratic equations. Testing the roots of the resulting equations as possible roots of the surd equations.

Examples in surd equations should be limited to those involving not more than two unlike surds, with or without a constant. The more complicated examples should be avoided.

Section 12. **Arithmetic and Geometric Series**

Fundamental theorems—nth term and the sum of n terms—and simple problems based on these formulae.

Arithmetic and geometric means.

Optional: The infinite geometric series.

GRADE 13 ALGEBRA

Section 1. Arithmetic and geometric series: an extension of the work as outlined for Grade 12 with the inclusion of the infinite geometric series: the sum of the squares of the natural numbers, and related series.

Supplementary topics: the sum of the cubes of the natural numbers; the arithmetic-geometric series; a brief account of the harmonic series including the definition of the harmonic mean between the numbers a and b and the relation holding among their arithmetic, geometric and harmonic means.

Section 2. Ratio, proportion and variation: an extension of the work outlined for Grade 12, to include the following:

$$\text{if } \frac{a}{b} = \frac{c}{d} \text{ then each of these} = \frac{a+c}{b+d} \text{ and,}$$

$$\text{more generally, } \frac{la+mc}{lb+md};$$

the solution of $ax + by + cz = 0$, $a_1x + b_1y + c_1z = 0$ to obtain the ratios of $x : y : z$, with direct applications; direct, inverse and joint variation (including the type $y = k_1 + k_2x$) reviewed to include fundamental theorems and exercises thereon.

Section 3. Functions: examples of functions defined by expressions, by statements, by graphs, by tables of values; formal definition of function.

Linear functions: review, including their graphical representation; determination of a linear function from two known values; the linear function in the direct variation specified above.

Section 4. Quadratic functions: review of the work outlined for Grade 12; the factoring of these functions when they possess real factors; the values of the variable which produce positive, zero, negative, maximum or minimum values for the function, together with the graph-

ical representation and interpretation of the same; review of quadratic equations with real roots, with emphasis on the meaning of a root of an equation.

Number systems: brief discussion of the systems of whole numbers, signed integers, rational numbers, real numbers, complex numbers, and the laws of combination of the numbers of these systems.

Quadratic functions continued: the factoring of quadratic functions; quadratic equations; the discriminant; its use in determining the maximum or minimum value of a quadratic function; the sum and product and other simple symmetric functions of the roots.

Section 5. Rational functions: definitions of integral and of fractional rational functions of one variable; statements of theorems on the existence of roots of algebraic equations; remainder theorem and factor theorem; determination of a quadratic function from three known values; simple fractional rational functions such as

$$\frac{1}{x}, \quad \frac{1}{x^2+1}, \quad \frac{x}{x^2+1},$$ their graphs and their properties;

decomposition of simple functions, e.g.

$$\frac{3x-5}{(x-1)\,(x-2)} \text{ into } \frac{2}{x-1} + \frac{1}{x-2};$$

solution of equations which depend upon quadratics and of equations for which a root may be discovered by inspection.

Supplementary topics: graphs of simple fractional rational functions, e.g. $\dfrac{3x-5}{(x-1)\,(x-2)}$; criteria of symmetry of a graph with respect to its reference axis; factoring by symmetry.

Section 6. Definition of integral and of fractional rational functions of more than one variable; systems of linear equations; systems of two equations of which one

is linear and one quadratic; simple systems of two quadratic equations.

Section 7. Permutations and combinations: the usual topics, omitting consideration of the value of r for which ${}_nC_r$ is greatest.

Section 8. The binomial theorem: its proof for positive integral indices, discussion for other real indices, with emphasis on sufficient conditions for the truth of the theorem; statement of a general term in any binomial expansion. Customary topics which may be omitted are: the recognition of the binomial from which a given series is derived, the properties of the coefficients, the numerically greatest term, the number of homogeneous products.

Section 9. Mathematics of investment: amount and present value of a sum of money and of an annuity; bonds, debentures, mortgages, sinking funds. In this topic the objective is an intelligent use of the geometric series rather than the development of formulae and symbols.

GRADE 13 GEOMETRY

The content of the course is as follows:

Section 1. **The Notion of Co-ordinates**

Axes of reference; Cartesian co-ordinates of a point.

Section 2. **First Applications of Co-ordinates**

Distance between points; internal and external division of a line segment in a given ratio; areas of triangles and other rectilinear figures in terms of the co-ordinates of the vertices.

Section 3. **Slope of a Line**

Definition; condition for parallelism and for perpendicularity of two lines.

Supplementary topic: angle between two lines.

Section 4. **Loci and their Equations**

Locus defined as the path traced by a point which moves according to a given law; the equation of the locus as the algebraic statement of this law; drawing simple loci by (1) interpreting their equations, (2) plotting a number of points; the points of intersection of loci.

Section 5. **The Straight Line**

Various forms of equation of line (slope and one point, slope and y-intercept, two points, two intercept); general equation of line; slope of line whose equation is given; conditions for parallelism and perpendicularity in terms of the coefficients; reduction of equation of line to desired form; families of lines.

Perpendicular distance from a point to a line; sign of $ax + by + c$ as (x, y) varies; bisectors of angle between two lines.

Supplementary topic: the normal form, and the direction cosine form, of the equation of a line; the equation of a pair of lines through the origin; the angle between these lines.

Section 6. **The Circle**

Equations of circle in standard forms; reduction of general equation of circle to standard form.

Definition of a tangent to a curve as the limit of a

secant; equation of tangent to circle at a given point; tangent with given slope; equation of tangent from a given point in numerical cases only; length of tangent; radical axis.

Families of circles.

Section 7. **The Parabola**

Definition of parabola—the locus of a point which is equidistant from a fixed point and a fixed line.

Construction by (1) point-by-point method using ruler and compasses, (2) mechanical method using set-square and cord.

Development of equation $y^2 = 4px$; determination, from this equation, of intercepts, range of values of coordinates, symmetry; the number of points of intersection of a parabola with an arbitrary straight line; latus rectum; drawing of parabolas from the equations in numerical cases by plotting points.

Discussion of the equation $y^2 = -4px$, $x^2 = 4py$, and $x^2 = -4py$ with attention to ranges of values and to symmetry.

Equations of tangent and normal at a given point; tangent with a given slope; tangent from a given point in numerical cases; lengths of subnormal and subtangent; construction of tangent at a given point by ruler and compasses.

Proof of theorem: the normal to a parabola at any point bisects the angle between the focal radius to the point and the line through the point parallel to the axis of the parabola. Application of this theorem to the construction of parabolic mirrors in searchlights and reflecting telescopes.

Diameters of a parabola.

Examples of the occurrence of the parabola in physics.

Supplementary topic: the locus of the intersection of perpendicular tangents to a parabola is the directrix.

Section 8. **The Ellipse**

Definition—based on the constant sum of the focal distances.

Construction by (1) point-by-point method using ruler and compasses, (2) mechanical method using cord.

Development of the equation $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$; determination, from this equation, of intercepts, range of values of co-ordinates, symmetry; the number of points of intersection of an ellipse with an arbitrary straight line; latus rectum; eccentricity.

Tangent and normal at a point on the ellipse; tangent of given slope; construction of tangent at a point on the ellipse by means of the circle on major (or minor) axis as diameter.

Diameters of an ellipse; conjugate diameters.

Determination of area of ellipse by comparison of ordinates of ellipse with those of circle on major (or minor) axis as diameter.

Supplementary topics: geometric construction of ellipse from circles on major and minor axes; formulae $a + ex$ and $a - ex$ for focal distances; tangent and normal at a point bisect the angles between focal radii; definition of ellipse in terms of focus, directrix and eccentricity; Kepler's Laws of Planetary Motion.

Section 9. **The Hyperbola**

Definition—based on constant difference of focal distances.

Construction by (1) point-by-point method using ruler and compasses, (2) mechanical method using cord.

Development of the equation $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$; determination, from this equation, of intercepts, range of values of co-ordinates, symmetry; the number of points of intersection of an hyperbola with an arbitrary straight line; latus rectum; eccentricity.

Discussion of the equation $y = \pm \frac{a}{b}\sqrt{x^2 - a^2}$ for large values of x; asymptotes.

Discussion of the equation $\frac{x^2}{b^2} - \frac{y^2}{a^2} = -1$;

conjugate hyperbola.

The equilateral hyperbola $x^2 - y^2 = a^2$; development of the equation $xy = \frac{a^2}{2}$ when the asymptotes are taken as axes of co-ordinates.

Examples of the occurrence of the equilateral hyperbola in physics.

Supplementary topics: application of hyperbolas to navigation and to the locating of an invisible source of sound; formulae $ex - a$ and $ex + a$ for focal distances; definition of hyperbola in terms of focus, directrix and eccentricity; tangents and normals; diameters and conjugate diameters.

GRADE 13 TRIGONOMETRY AND STATICS

Trigonometry

In this subject two objectives are to be kept in mind. One of these is the solution of triangles, a problem fundamental in surveying, the other is the development of analytic trigonometry, that is, the theory of the circular functions, as outlined in sections 3, 5, 8 and 10 of the following syllabus. It is to be noted that the first of these objectives can be accomplished independently of the second. All the formulae necessary for the solution of triangles, with or without the use of logarithms, can be obtained from geometry and the definitions of the trigonometric functions. Hence, if desired, the topics of sections 6 and 7 may be studied before those of section 5. The teacher should not spend much time on the solution of triangles without logarithms.

Section 1. Review of the trigonometry of previous grades with attention to the solution of right-angled triangles and applications; values of the trigonometric functions of 30°, 45° and 60°.

Section 2. Definitions of the trigonometric functions of positive and of negative angles of any size (these may be given in terms of the co-ordinates of a point on the unit circle); signs of the functions of angles belonging to each of the four quadrants; values of functions for such angles as 135°, 210°, found from diagrams. Proofs of the following formulae where A is an angle of any magnitude:

$$\tan A = \frac{\sin A}{\cos A};\ \cos^2 A + \sin^2 A = 1;\ \sec^2 A = 1 + \tan^2 A;$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A.$$

Section 3. Variation of the trigonometric functions. The values of the functions of 0°, 90°, 180°, 270°, where these values exist; the changes in sign and magnitude of sin x, cos x, tan x, as x increases from 0° to 360°; the periodicity and the graphs of sin x, cos x, tan x; the functions of $-$ x, (90° $\pm$ x), (180° $\pm$ x), (270° $\pm$ x), found from diagrams in terms of the functions of x. (Omit longer identities on related angles.)

Section 4. Review logarithms. Definition and under-

standing of $\log_a x$; range of values of x for which $\log_a x$ is defined; graph of $\log_a x$. Working rules of logarithms:

$$\log_a xy = \log_a x + \log_a y; \quad \log_a \frac{x}{y} = \log_a x - \log_a y;$$

$\log_a x^p = p \log_a x$. Computations with logarithms. Solution of exponential equations in one unknown.

Supplementary topic: the slide rule.

Section 5. Functions of the sum and of the difference of two angles (addition formulae). Sine and cosine of (A + B) and of (A − B), (proofs for the cases in which A, B, A + B, A − B are positive acute angles; discussion also for angles of any magnitude; derivation of tan (A + B) and of tan (A − B); values of the functions for such angles as 15°, 75°; practice in such relations as

$$\sin (A + B) = \sin C \text{ and } \tan \frac{A+B}{2} = \cot \frac{C}{2} \text{ where}$$

$A + B + C = 180°$; functions of 2A in terms of those of A and, correspondingly, of θ in terms of $\frac{\theta}{2}$; transformations of sums and differences into products and of products into sums and differences.

Supplementary topic: the functions of 3A in terms of those of A, with application to the ratios of 18° and its multiples.

Section 6. Some properties of triangles. The law of sines $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$; the law of cosines $\cos A = \frac{b^2 + c^2 - a^2}{2bc}$; the relation $a = b \cos C + c \cos B$; application of these laws to the solution of triangles in suitable cases, and an understanding of the practical limitations in the case of the cosine law.

Section 7. The solution of triangles with the aid of logarithms. Formulae adapted to logarithms; the law of sines; the functions of the half-angles in terms of the sides (geometric or analytic proofs); the law of tangents,

$$\tan \frac{B-C}{2} = \frac{b-c}{b+c} \cot \frac{A}{2}$$ (geometric or analytic proof);

application of these formulae to the solution of triangles; problems in heights and distances in one or more planes. (Those in more than one plane should be confined to simple, direct examples.)

Supplementary topic: identities based on formulae for oblique triangles.

Section 8. Radian measure.

The relation $\frac{\text{circumference}}{\text{diameter}} = \text{constant}$; symbol for this constant and its approximate value. Definition of radian; radian measure of any angle $(\theta = \frac{a}{r})$; the formulae $a = r\theta$ and 180 degrees $= \pi$ radians.

Section 9. Further properties of triangles. Area of triangle; radius of circumscribed circle; radii of inscribed and escribed circles ($r = \frac{\Delta}{s}$ and $r = (s-a) \tan \frac{A}{2}$ with corresponding formulae for r_1, r_2, r_3); area of circle; area of sector of a circle.

Supplementary topic: perimeter and area of regular polygon, (i) inscribed in a circle, (ii) circumscribed about a circle.

Section 10. Solution of easy trigonometric equations, limiting the roots to values lying between $+ 360°$ and $- 360°$.

Supplementary topics: Inverse circular functions. Meaning and simple evaluations of functions $\sin^{-1} x$,

$\cos^{-1} x$, $\tan^{-1} x$; general solution for trigonometric equations involving the sine, cosine and tangent.

Statics

It is expected that wherever possible experimental demonstrations will accompany the teaching of the various topics.

1. Weight: the equal-arm balance; bodies of equal weight; the weight of a body in terms of the weight of a standard body.

2. Force as a concept associated with our sensations when lifting or pushing a body.

   Point of application, direction, and magnitude as elements of a force; vector representation of a force.

   The weight of bodies, at a place on the earth, as forces and measures of forces.

   The adequacy or inadequacy of the weight of a standard body as the unit of force.

   The spring balance: its calibration and its use in the measurement of force.

   The transmission of force by a string under tension and by a rod under compression.

3. Experimental study of the composition and resolution of forces pushing or pulling a particle. The parallelogram law.

   Calculation of components and resultants.

   Resultant of two collinear and of two, three, or more coplanar but non-collinear forces acting on a particle.

   The triangle of forces.

   The conditions of equilibrium of a particle acted upon by two, three, or any number of forces.

   Supplementary topic: the polygon of forces.

4. The capacity of a solid body to transmit force with preservation of direction.

   Experimental study of levers, leading to the definition of the moment of a force about a point.

   The equality of the sum of the moments of two or more forces and the moment of their resultant.

   The composition of parallel forces. Couples.

Centre of gravity: its determination by experiment and, for bodies of simple shape, by calculation.

5. Newton's Third Law—when one body exerts a force on a second, the second exerts a force on the first equal in magnitude and opposite in direction.

6. The reduction of a set of coplanar forces on a lamina to a simple set of forces.
   The conditions for the equilibrium of a lamina.

7. Friction: illustrative examples; laws of friction; coefficient of friction.
   Direction of the reaction of a smooth surface.

Supplementary Topic:

Machines: mechanical advantage; lever, pulley, differential pulley, wheel and axle, train of toothed wheels, inclined plane, wedge, screw.

Suggestions for Teachers: Calculations with Approximate Numbers

A. *The importance of the concepts underlying calculations based on measurement*

It is to be emphasized that computation with approximate numbers should not be thought of as an isolated topic in Trigonometry and Statics or in Grade 13 Physics. On the contrary, clear ideas regarding the nature and accuracy of numbers obtained by measurement and the methods to be used in calculations involving such numbers should gradually be built up in the pupils' minds as they progress from Grade 9 to Grade 13. Suitable opportunities arise whenever quantitative experiments are performed or problems involving measured numbers are attempted. If this procedure is followed no difficulty will be experienced in attaining the proficiency indicated in the following section.

B. *Minimum knowledge expected of a Grade 13 student regarding operations with numbers*

1. The student should have come to realize clearly, from experimental work in Mathematics and Science in the earlier grades, that all numbers obtained by measurement are inexact. He should also have a reasonably good idea of the accuracy to be expected under given conditions of measurement.

2. He should know the meaning of the terms "possible error", "precision", "relative error", "percentage error", "relative accuracy", "significant digits", and of such expressions as "correct to four significant digits", "correct to four decimal places", "correct to the nearest minute".

3. He should be familiar with the principle governing the precision of the results obtained in the addition and subtraction of approximate numbers and with the fact that the precision of the result depends on that of the least precise of the given numbers. He should also be familiar with the principle governing the accuracy of results obtained in the multiplication and division of approximate numbers and with the fact that the accuracy of the result depends on that of the least accurate of the data.

Thus, in operations involving addition and subtraction

of approximate numbers of which the least precise has n places of decimals, the pupil should be taught to round off other numbers, where possible, to $n + 1$ places and the answer to n places. Similarly, in operations involving multiplication and division of approximate numbers of which the least accurate has n significant digits, he should round off the others, where possible, to $n + 1$ digits and the answer to n digits.

A Grade 13 pupil should also be acquainted with the procedure for finding the upper and lower limits in typical calculations and should understand that such a procedure is employed where a highly precise answer is necessary.

4. In the solution of triangles, he should be familiar with the relation between the degree of accuracy of linear data and that of the angles calculated from them—"two-place" data corresponding in general to angles correct to the nearest degree.

5. In calculations with logarithms he should understand that, in order to obtain a result correct to n significant digits, he must have data of at least this accuracy. Further, he should use logarithms with mantissas of at least $n + 1$ decimal places, rounding off the final answer to n significant digits.

6. The candidate should be made aware of the fact that in the Departmental examinations he will be required to use five-figure tables. He must, of course, know how to use the mean differences shown and should also be able to perform any necessary interpolations.

C. *Credits and Penalties for "Rounding Off" as applied in recent annual Departmental Examinations*

1. Normally, one mark is lost for not properly rounding off the answer in each problem in which a specified degree of accuracy is required. Thus, a candidate who is ignorant of the principles involved may lose several marks in the aggregate.

2. There is no penalty for using five-place logarithms throughout the work in any problem which could properly be handled with tables to fewer places, provided the answer is properly rounded off to express the required degree of accuracy.

3. On the other hand, there is no penalty for using less than five-place mantissas, properly rounded off, provided that one more decimal place is carried in the work than the number of significant digits required in the answer.

4. In problems involving computation there is ordinarily a penalty of one mark per question for (i) Rounding off too soon: that is, failure to use where possible throughout the work, one more digit from the tables than is given in the data and required in the answer, (ii) Rounding off too late: that is, the repeated use of 6 or more decimal places in handling five-place logarithms. Such apparent accuracy is spurious and such procedure is penalized as "bad form".

D. *Supplementary notes and examples*

1. "When the numbers used in problems have ambiguous zeros, the author should always state the unit or the number of significant digits, thus removing the ambiguity".*

Thus, if the unit of measurement in 7200 is 100, either this should be stated, or the fact of two significant digits should be mentioned, or the number should be written 72×10^2. If 7200 is written without such qualification, one is justified is assuming all four digits to be significant.

2. An illustration from Grade 9:

On their first attempt at metric measurement, six pupils turn in the following results for the length of their classroom as measured with a metric stick: 9 metres, 87 cm; 10 metres; 10.8 metres; 9 metres 9 dcm; 9.836 metres; 9.79 metres. Required, the best approximation warranted by these measurements.

(*a*) Obviously 10.8 indicates a careless mistake and 10 a degree of precision below that easily possible; both should be eliminated.

(*b*) Rounding each of the remaining four answers to two places of decimals, adding, and rounding the sum to one place, we get 39.4 (The error here will not exceed 0.1) Arithmetic mean of the four results is 9.850 which is rounded off to 9.85 — a suitable result.

3. In Grade 10 Practical Geometry, pupils have been given the dimensions of a triangle ABC, (base $a = 9.0$ cm.) and have measured the altitude. The class average for the latter has been determined as 7.6 cm. What is the area?

$$\text{Area (in sq. cm.)} = \frac{9.0 \times 7.6}{2} = \frac{68.40}{2} = 34.20$$

Rounded off to two significant figures, the result is 34 sq. cm.

4. The following problem is presented in Grade 11 Physics:

A mass of 300 gm. of water at 10°C is contained in a copper calorimeter weighing 100 gm. If 500 gm. of lead shot at 90°C is poured into the calorimeter the resulting temperature of the mixture is 14°C. Calculate the specific heat of the lead. (Specific heat of Cu = 0.094) The resulting equation, where S is required specific heat,

$$500.S.\ (90—14) = 300 + 100 \times 0.094)\ (14 - 10) \text{ gives}$$

$$S = \frac{309.4 \times 4}{76 \times 500} = .033$$

Such a question can form the basis of a very valuable discussion on sources of error — loss of heat, precision of measurements, etc. Here the dominant factor in the accuracy of the answer is evidently the temperature difference of 4 degrees. Assuming that the temperatures have been read to the nearest degree, this factor introduces a possible error in the result of 1 degree or 25% (14.5 — 9.5 = 5; 13.5 — 10.5 = 3; 4° ± 1° as temperature difference). Hence the final digit in .033 is not significant.

Note: The quotation marked* in D. 1. above is from *COMPUTATIONS WITH APPROXIMATE NUMBERS*, by William A. Gager, in "School Science and Mathematics", May 1947.